Homework 9

1. Section 17

17.1 Let $f(x) = \sqrt{4-x}$ for $x \leq 4$ and $g(x) = x^2$ for all $x \in \mathbb{R}$.
(a) Give the domains of $f+g$, fg, $f \circ g$ and $g \circ f$.
(b) Find the values $f \circ g(0)$, $g \circ f(0)$, $f \circ g(1)$, $g \circ f(1)$, $f \circ g(2)$ and $g \circ f(2)$.
(c) Are the functions $f \circ g$ and $g \circ f$ equal?
(d) Are $f \circ g(3)$ and $g \circ f(3)$ meaningful?

17.3 Accept on faith that the following familiar functions are continuous on their domains: $\sin x$, $\cos x$, e^x, 2^x, $\log_e x$ for $x > 0$, x^p for $x > 0$ [p any real number]. Use these facts and theorems in this section to prove the following functions are also continuous.
(a) $\log_e(1 + \cos^4 x)$
(e) $\tan x$ for $x \neq$ odd multiple of $\frac{\pi}{2}$. (f) $x \sin \frac{1}{x}$ for $x \neq 0$.

17.4 Prove the function $\sqrt{x}$ is continuous on its domain $[0, \infty)$. *Hint*: Apply Example 5 in Section 8.

17.6 A rational function is a function f of the form p/q where p and q are polynomial functions. The domain of f is $\{x \in \mathbb{R} : q(x) \neq 0\}$. Prove every rational function is continuous. *Hint*: Use Exercise 17.5.

17.7 (a) Observe that if $k \in \mathbb{R}$, then the function $g(x) = kx$ is continuous by Exercise 17.5.
(b) Prove $|x|$ is a continuous function on $\mathbb{R}$.
(c) Use (a) and (b) and Theorem 17.5 to give another proof of Theorem 17.3.

17.9 Prove each of the following functions in continuous at x_0 by verifying the $\varepsilon - \delta$ property of Theorem 17.2.
(a) $f(x) = x^2$, $x_0 = 2$;
(b) $f(x) = \sqrt{x}$, $x_0 = 0$;
(c) $f(x) = x \sin(\frac{1}{x})$for $x = 0$ and $f(0) = 0$, $x_0 = 0$;

17.10 Prove the following functions are discontinuous at the indicated points. You may use either Definition 17.1 or the $\varepsilon - \delta$ property in Theorem 17.2.
(a) $f(x) = 1$ for $x > 0$ and $f(x) = 0$ for $x \leq 0$, $x_0 = 0$;
(b) $g(x) = \sin(\frac{1}{x})$ for $x = 0$ and $g(0) = 0$, $x0 = 0$;
(c) $\text{sgn}(x) = -1$ for $x < 0$, $\text{sgn}(x) = 1$ for $x > 0$, and $\text{sgn}(0) = 0$, $x_0 = 0$. The function sgn is called the *signum function*; note $\text{sgn}(x) = \frac{x}{|x|}$ for $x = 0$.

17.12 (a) Let f be a continuous real-valued function with domain (a, b). Show that if $f(r) = 0$ for each rational number r in (a, b), then $f(x) = 0$ for all $x \in (a, b)$.
(b) Let f and g be continuous real-valued functions on (a, b) such that $f(r) = g(r)$ for each rational number r in (a, b). Prove $f(x) = g(x)$ for all $x \in (a, b)$. *Hint*: Use part (a).

17.13 (a) Let $f(x) = 1$ for rational numbers x and $f(x) = 0$ for irrational numbers. Show f is discontinuous at every x in $\mathbb{R}$.
(b) Let $h(x) = x$ for rational numbers x and $h(x) = 0$ for irrational numbers. Show h is continuous at $x = 0$ and at no other point.