Homework 7

1. Section 14

14.1 Determine which of the following series converge. Justify your answers.
(a) $\sum \frac{n^4}{2^n}$
(d) $\sum \frac{n!}{n^4+3}$
(e) $\sum \frac{\cos^2 n}{n^2}$

14.2 Repeat Exercise 14.1 for the following.
(a) $\sum \frac{n-1}{n^2}$
(e) $\sum \frac{n^2}{n!}$
(g) $\sum \frac{n}{2^n}$

14.3 Repeat Exercise 14.1 for the following.
(a) $\sum \frac{1}{\sqrt{n!}}$
(c) $\sum \frac{1}{2^n+\sqrt{n}}$
(e) $\sum \sin(\frac{n\pi}{9})$

14.4 Repeat Exercise 14.1 for the following.
(a) $\sum \frac{1}{(n+(-1)^n)^2}$
(b) $\sum[\sqrt{n+1}-\sqrt{n}]$

14.6 (a) Prove that if $\sum |a_n|$ converges and (b_n) is a bounded sequence, then $\sum a_n b_n$ converges.
Hint: Use Theorem 14.4.
(b) Observe that Corollary 14.7 is a special case of part (a).

14.7 Prove that if $\sum a_n$ is a convergent series of nonnegative numbers and $p > 1$, then $\sum a_n^p$ converges.

14.8 Show that if $\sum a_n$ and $\sum b_n$ are convergent series of nonnegative numbers, then $\sum \sqrt{a_n b_n}$ converges.
Hint: Show $\sqrt{a_n b_n} \leq a_n + b_n$ for all n.

4.10 Find a series $\sum a_n$ which diverges by the Root Test but for which the Ratio Test gives no information. Compare Example 8.

14.12 Let $(a_n)_{n\in\mathbb{N}}$ be a sequence such that $\liminf |a_n| = 0$. Prove there is a subsequence $(a_{n_k})_{k\in\mathbb{N}}$ such that $\sum_{k=1}^{\infty} a_{n_k}$ converges.

14.14 Prove $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by comparing with the series $\sum_{n=2}^{\infty} a_n$ where (a) is the sequence

$$(\tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{8}, \tfrac{1}{8}, \tfrac{1}{8}, \tfrac{1}{8}, \tfrac{1}{16}, \tfrac{1}{16}, \tfrac{1}{16}, \tfrac{1}{16}, \tfrac{1}{16}, \tfrac{1}{16}, \tfrac{1}{16}, \tfrac{1}{16}, \tfrac{1}{32}, \tfrac{1}{32}, \ldots.)$$