Homework 6

1. Section 11

11.8 Use Definition 10.6 and Exercise 5.4 to prove $\liminf s_n = -\limsup(-s_n)$ for every sequence (s_n).

11.9 (a) Show the closed interval $[a, b]$ is a closed set.
(b) Is there a sequence (s_n) such that $(0, 1)$ is its set of subsequential limits?

2. Section 12

12.2 Prove $\limsup |s_n| = 0$ if and only if $\lim s_n = 0$.

12.4 Show $\limsup(s_n + t_n) \leq \limsup s_n + \limsup t_n$ for bounded sequences (s_n) and (t_n). *Hint*: First show $\sup\{s_n + t_n n > N\} \leq \sup\{s_n : n > N\} + \sup\{t_n : n > N\}$. Then apply Exercise 9.9(c).

12.6 Let (s_n) be a bounded sequence, and let k be a nonnegative real number.
(a) Prove $\limsup(ks_n) = k \cdot \limsup s_n$.
(b) Do the same for $\liminf$. *Hint:* Use Exercise 11.8.
(c) What happens in (a) and (b) if $k < 0$?

12.8 Let (s_n) and (t_n) be bounded sequences of nonnegative numbers. Prove $\limsup s_n t_n \leq (\limsup s_n)(\limsup t_n)$.

12.9 (a) Prove that if $\lim s_n = +\infty$ and $\liminf t_n > 0$, then $\lim s_n t_n = +\infty$.
(b) Prove that if $\limsup s_n = +\infty$ and $\liminf t_n > 0$, then $\limsup s_n t_n = +\infty$.
(c) Observe that Exercise 12.7 is the special case of (b) where $t_n = k$ for all $n \in \mathbb{N}$.

12.10 Prove (s_n) is bounded if and only if $\limsup |s_n| < +\infty$.

12.11 Prove the first inequality in Theorem 12.2.