Homework 5

1. Section 11

11.2 Consider the sequences defined as follows:
$$a_n = (-1)^n, \quad b_n = \frac{1}{n}, \quad c_n = n^2, \quad d_n = \frac{6n+4}{7n-3}.$$

(a) For each sequence, give an example of a monotone subsequence.
(b) For each sequence, give its set of subsequential limits.
(c) For each sequence, give its $\lim \sup$ and $\lim \inf$.
(d) Which of the sequences converges? diverges to $+\infty$? diverges to $-\infty$?
(e) Which of the sequences is bounded?

11.3 Repeat Exercise 11.2 for the sequences:
$$s_n = \cos\left(\frac{n\pi}{3}\right), \quad t_n = \frac{3}{4n+1}, \quad u_n = \left(-\frac{1}{2}\right)^n, \quad v_n = (-1)^n + \frac{1}{n}.$$

11.6 Show every subsequence of a subsequence of a given sequence is itself a subsequence of the given sequence. *Hint*: Define subsequences as in (3) of Definition 11.1.

2. Supplement Homework

S1. Give an example of each of the following, or prove that such a request is impossible.

(a) A sequence that does not contain 0 or 1 as a term but contains subsequences converging to each of these values.
(b) A monotone sequence that diverges but has a convergent subsequence.
(c) A sequence that contains subsequences converging to every point in the infinite set $\{1, 1/2, 1/3, 1/4, 1/5, \ldots\}$.
(d) An unbounded sequence with a convergent subsequence.
(e) A sequence that has a subsequence that is bounded but contains no subsequence that converges.