

PROBLEM SET FOR DAY 2
NOTES FOR THE OREGON SUMMER SCHOOL 2013

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Topics:

- The Hilbert manifold $\Omega K = \text{Map}_\bullet(S^1, K)$ of smooth based loops into a compact group, and its Kahler structure.
- Morse-Bott theory of the energy functional.
- Presentation of the homology groups.
- The lattice model of the affine Grassmannian for $U(k)$.
- The affine flag manifold and its finite-dimensional and finite-codimensional Schubert varieties.
- Affine Bruhat order.
- k -double Schur functions.

(*) = preferred problems

Consider ΩK as LK/K . Figure out what the left-invariant symplectic structures are.

Show that the symplectic gradient of the energy functional generates the loop rotation.

(*) If $K = U(1)$, describe the Morse-Bott strata as explicitly as possible, and why their union isn't all of ΩK .

(*) Show that each Schubert variety on the affine Grassmannian for $U(n)$ can be identified with a Springer variety on a finite-dimensional Grassmannian (i.e. a component of the fixed points of a nilpotent).

(*) Let $\lambda = (\lambda_1 \geq \dots \geq \lambda_n) \in \mathbb{N}^n$ be a partition of n , and \mathcal{O}_λ the corresponding nilpotent orbit in $\mathfrak{gl}(n)$ (the matrices with that JCF). Show that the Schubert variety X^λ in Gr for $U(n)$ contains an open set isomorphic to \mathcal{O}_λ [Lusztig].

Compute the minimal representative in the coset $t_\lambda W \in W_{\text{aff}}/W$, where λ is a coweight.

Show that for any element $v \in W_{\text{aff}}$, there is an antidominant coweight μ such that vt_μ is a minimal coset representative.

Compute the k -double Schur functions for $SL(2)$.