Let W be a finite group and let V^* be a graded representation of W. We say that V^* is **equivariantly log concave** if, for every i, $V^i \otimes V^i$ contains $V^{i-1} \otimes V^{i+1}$ as a subrepresentation. We say that V^* is **strongly equivariantly log concave** if, whenever $i \leq j \leq k \leq l$ and $i + l = j + k$, $V^j \otimes V^k$ contains $V^i \otimes V^l$ as a subrepresentation. These notions were introduced in [GPY17, §5]. When W is trivial, these two notions are equivalent. For general W, the notion of strong equivariant log concavity is more robust because it is preserved by taking tensor products, whereas ordinary equivariant log concavity is not.

Let M be a matroid equipped with an action of a finite group W, which then acts on the Orlik-Solomon algebra OS_M^*. The following conjecture appears in [GPY17, 5.3].

Conjecture 1. *The Orlik-Solomon algebra* OS_M^* *is strongly equivariantly log concave.*

Remark 2. The graded ring OS_M^* has Poincaré polynomial equal to the characteristic polynomial of M. That means that, when W is trivial, Conjecture 1 coincides with the theorem of Adiprasito, Huh, and Katz on log concavity of the characteristic polynomial [AHK18].

Remark 3. An interesting special case is where M is the braid matroid of rank $n-1$ and W is the symmetric group S_n. Then OS_M^* is isomorphic to the cohomology of the configuration space of n distinct labeled points in $\mathbb{R}^2$.

Remark 4. There are many variants of Conjecture 1. For example, if $\mathcal{A}$ is a hyperplane arrangement with an action of W, we can consider the Artinian Orlik-Terao algebra $\mathrm{AO}_{\mathcal{A}}^*$, which carries an action of W. This is isomorphic as a graded vector space to the Orlik-Solomon algebra of the associated matroid, but the isomorphism cannot be made equivariant. For example, in the case of the braid arrangement, the Artinian Orlik-Terao algebra is isomorphic to the cohomology of the configuration space of n distinct labeled points in $\mathbb{R}^3$ (with degrees halved). We also conjecture that $\mathrm{AO}_{\mathcal{A}}^*$ is strongly equivariantly log concave.

Remark 5. The S_n-equivariant log concavity property for the cohomology rings $H^*(\mathrm{Conf}(n, \mathbb{R}^2); \mathbb{C})$ and $H^*(\mathrm{Conf}(n, \mathbb{R}^3); \mathbb{C})$ alluded to in Remarks 3 and 4 has been checked on a computer up to $n = 10$.

References

[AHK18] Karim Adiprasito, June Huh, and Eric Katz, *Hodge theory for combinatorial geometries*, Ann. of Math. (2) **188** (2018), no. 2, 381–452.

[GPY17] Katie Gedeon, Nicholas Proudfoot, and Benjamin Young, *The equivariant Kazhdan–Lusztig polynomial of a matroid*, J. Combin. Theory Ser. A **150** (2017), 267–294.