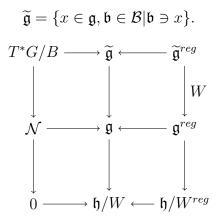
TRIAGE

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Recall the Grothendieck-Springer resolution:



The Beilinson-Bernstein theorem gives an equivalence

$$\mathcal{D}(G/B) \stackrel{\Gamma}{\underset{\Delta}{\leftrightarrow}} U\mathfrak{g}_0 \operatorname{-mod}.$$

Both categories carry smooth G-actions, i.e. they are module categories over $(\mathcal{D}(G), *)$ (a monoidal dg-category).

 $\mathcal{D}(G/B)$ carries a left module structure over $\mathcal{D}(G)$ and a right module structure over $\mathcal{D}(B \setminus G/B) =: \mathcal{H}$. In fact, End $\mathcal{D}(G/B) = \mathcal{D}(B \setminus G/B)$.

Recall from the first talk that $\mathcal{H}(G, K)$ -mod is equivalent to $\mathbb{C}G$ -mod generated by their K-invariants, i.e. $\mathbb{C}G$ -mod generated by $\mathbb{C}[G/K]$.

Similarly, \mathcal{H} -mod is equivalent to the subcategory of $\mathcal{D}(G)$ -mod generated by $\mathcal{D}(G/B)$.

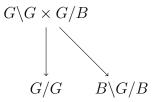
Let X be a variety with a G-action. Then $\mathcal{D}(X)^K = \mathcal{D}(K \setminus X)$, which carries an action of $\mathcal{H}(G, K)$.

We get that $\mathcal{D}(K \setminus G/B) = \mathcal{D}(G/B)^K$. By Beilinson-Bernstein we get $\mathcal{D}(G/B)^K \cong (U\mathfrak{g}_0 \operatorname{-mod})^K$, which can be identified with (\mathfrak{g}, K) -mod.

We can identify the category \mathcal{O} with

$$\mathcal{O}_0 = \mathcal{D}(B \backslash G/B),$$

where the right *B*-action gives a strongly *N*-equivariant \mathcal{D} -module, which is weakly *H*-equivariant.



The space on the top can be identified with G/B, where B acts by conjugation. It has a subspace $B/B \cong \widetilde{G}/G$. We get

$$\pi_*(\mathbf{C}_{\widetilde{G}/G}) \subset D(G/G),$$

the Grothendieck-Springer sheaf.

To construct a character sheaf, take $\mathcal{D}_{G/G}$. To make the characteristic variety sit inside the nilpotent cone, mod out $\mathcal{D}_{G/G}$ by the set of all *G*-biinvariant differential operators on *G*. In this way we get the Harish-Chandra sheaf.

Theorem (Hotta-Kashiwara). The Grothendieck-Springer sheaf coincides with the Harish-Chandra system sheaf.