1. Compute all the c_{SM} classes of Schubert cells for $G = GL_3$, both in terms of the monomial basis and the Schubert basis.

2. Consider $w = 3412 \in S_4$.

   (a) Describe the linear algebra conditions on the flag to belong to X_w° and X_w. (Optional: Figure out what the minimal set of conditions are for belonging to X_w.)

   (b) Find a reduced decomposition for w.

   (c) List all $v \leq w$ in Bruhat order.

   (d) Calculate the class $[X_w]$ in terms of the x_i's.

   (e) Draw the diagram of the Bott-Samelson resolution for w

   (f) Describe the fiber (including what the choice for the extra subspaces must be in linear algebraic terms) for an arbitrary point in the Schubert cell.

   (g) Describe the fiber over the point corresponding to a flag of the form $E_1 \subseteq F_2 \subseteq E_3$, where E_1 and E_3 are the standard subspaces and F_2 is any two dimensional subspace containing E_1 and contained in E_3.

   (h) Describe the B-orbits of the Bott-Samelson. (Hint: There was a reason for the last part.)

   (i) Describe the T-fixed points of the Bott-Samelson. Let $U_t = \text{diag}(1, t, t^2, t^3)$, and for each T fixed point p, describe the set $\{x \mid \lim_{t \to 0} U_t \cdot x = p\}$.

3. Consider the Schubert variety in the Grassmannian $G(3, 5)$ given by the partition $(2, 1, 1)$ or the maximal coset representative $w = 53241$ or minimal coset representative $w = 23514$. Describe the two Zelevinsky resolutions. Find their B-orbits. Find their fibers.

4. Prove that the Bott-Samelson resolution is bijective over the Schubert cell. (Hint: You might want to induct on the length of your reduced expression.) For extra challenge, try doing this without explicitly using the fact that the Bott-Samelson resolution is B-equivariant.