Ben Elias Stress some key points: Oregon 8/2014 Tuesday Triage 1

① **Algebraic Replacement** $\mathcal{C}$ abelian cat, can be very hard to work with. Ex: $\mathcal{O}_0$

Let P be a projective generator, i.e. $\mathrm{Hom}(P,M)=0 \Rightarrow M=0$. Then

$$\mathrm{Hom}(P,\cdot) : \mathcal{C} \xrightarrow{\sim} \mathrm{End}(P)^{op}\text{-mod}$$ so $\mathcal{C}$ can be expressed as modules over some ring.

Usual projective generator – $P_{min} = \bigoplus P_w$ sum of all indecomp. projectives. Certainly, it works...

but as we've seen in $\mathcal{O}$, finding P_w is hard, let alone $\mathrm{End}(P_{min}) = \bigoplus \mathrm{Hom}(P_w, P_v)$.

But category $\mathcal{O}$ has a combinatorial proj generator, $P_{BS} = \bigoplus_{\underline{w}} P(\underline{w})$ $P(\underline{w}) = \theta_{s_1} \cdots \theta_{s_2}\theta_{s_3} \cdot P_1$

not just reduced!

VERY redundant. But today, we show that $\mathrm{End}(P_{BS})$ is actually easier to describe than $\mathrm{End}(P_{min})$

We do it by generators + relations!!!

Well, OK, we do it with $\mathbb{S}Bim$, not $\mathcal{O}$, but it's the same idea.

$\cup$ BS Bim $\cup$ BS Proj

Algebraic Replacement

Redundancy helps – maps b/w red exp factor.

Same idea $\rightsquigarrow$ KLR algebras, Webster algebras in ~~...~~ catfn of quantum groups

Even before ability to describe by gen+relations, Soergel got tons of mileage out of restricting to BS Proj. It helps when you can work w/ objects having an easy description!!

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② Soergel Conjecture $[B_w] = \underline{H}_w$

Spose $\exists B$ (not necessarily indecomp) s.t. $[B] = \underline{H}_w$.

$$\underset{SHF}{\Longrightarrow} \mathrm{gdim}\,\mathrm{End}(B) = 1 + v\mathbb{Z}[v] \Longrightarrow \mathrm{End}(B) \text{ is local} \Rightarrow B \text{ is indecomp.}$$

$$\Rightarrow B = B_w \Rightarrow [B_w] = B$$

$$BS(\underline{w}) = B_w \oplus \bigoplus B_y^{\oplus m_y}$$ if you can show $m_y = \mu_y$ then you win!

rex

$$BS(\underline{w}) = \oplus B_y^{\oplus \mu_y} \oplus B$$ leftover

$$\underline{H}(\underline{w}) = \underline{H}_w + \sum \underline{H}_y \mu_y \qquad \mu_y, m_y \in \mathbb{Z}[v^{\pm}]$$

$[B] = \underline{H}_w$.

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③ KL presentation

$$\mathbb{H}_W = \langle H_s \rangle \big/ \begin{array}{c}(H_s+v)(H_s-v^{-1})=0 \\ H_sH_t\cdots = H_tH_s\cdots\end{array}$$

$$\mathbb{H}_W = \langle \underline{H}_s \rangle \big/ \begin{array}{c}\underline{H}_s\underline{H}_s = (v+v^{-1})\underline{H}_s \\ \underline{H}_s\underline{H}_t\cdots = \underline{H}_t\underline{H}_s\cdots \\ + \cdots \quad + \cdots \\ \text{lower terms}\end{array}$$

better suited for catfn.