

Many many parts.

$F_x \oplus F_{s_1, \dots, s_d}$ for rex x . I.e. $F_x^j \oplus \bigoplus_{y=x+j} BS(y)(j)$

Thm (Diagonal miracle): $F_x^j = \bigoplus B_z(j)$ (ie F_x is perverse) and $F_x^0 = B_x$.

All the non-perverse stuff is cancelled by homotopy.

So $F_x^j(-j) = \bigoplus B_z(0)$ could conceivably have hL, HR for $L = \emptyset$.

Today - $Rohr(x) := F_x^j(-j)$ has hL, HR for L , w/ form induced from $\bigoplus BS(y)(j)$

Note: This is stronger than HR(z) for each term $B_z \oplus F_x^j(-j) \dots B_z \oplus BS(y)$ but not necessarily as top summand!! In other words, $BS(y)$ is a big, non-~~semismall~~ ^{semismall} guy

\langle, \rangle is nondey but \langle, \rangle_L is typically not (never when not ~~semismall~~ ^{semismall})

\langle, \rangle pairs summands $B_w(-k)$ against $B_w(+k)$, but \langle, \rangle_L restricts to zero.

(actually, not quite. exercise showed $\langle, \rangle_L |_{B_w(+k)} = 0$ but $\langle, \rangle |_{B_w(-k)}$

can be nonzero.) Nonetheless, $\langle, \rangle_L |_{B_w(0)}$ is nice and nondegenerate (hL, HR) for all summands, not just the top one.

We'll get $Rohr(x)$ by induction, writing $F_x \oplus F_y F_s$ for $x = ys > y$.

Then, similarly, we'll use analogous arguments to study $F_x F_s$. ($B_x \rightarrow F_x \rightarrow \dots$)

First few hom degrees are

$$B_x B_s \rightarrow \boxed{F_x B_s \oplus B_x(1)} \rightarrow \dots \quad (B_s \rightarrow F_s \rightarrow \dots)$$

↑ serves as weak left exact substitute.

factorization for L on $B_x B_s$.

We'll use this to get $HR(x; s)$, $hL(x; s)$, and $HR(x; s)_s$ for all $s \geq 0$.